Study Guide

Module 3: Multiplying and Factoring Whole Numbers

In Module 2 we showed how place value gave us a very convenient way for finding the sum of two or more numbers. Very often we are required to add a number to itself several times. For example, we might want to know how much 3 pencils cost if each pencil costs 17¢. In this case we'd get the answer by computing 17 + 17 + 17. In essence, we'd be counting by 17's. The numbers we get as sums when we count by 17's are called the *multiples of* 17. In this module we shall learn how to use place value to compute multiples of whole numbers in a convenient way. The reason we want to do this is that it is inconvenient to compute large multiples of a number by using the addition methods of Module 2. For example, if we wanted to find the cost of 258 pencils at 17¢ each, we'd have to compute the 258th multiple of 17. That is, we'd have to count by 17's to get: 17, 34, 51, 68, 85, 102, . . . until we got to the 258th number in the listing. In this module we shall describe a more convenient way of getting the answer by a process called *multiplication*.

In place value notation there is a special way of indicating that we want, for example, the 258th multiple of 17. We write:

$$17 \times 258$$

which we read as "17 *times* 258". The answer is called the *product* of 17 and 258, while 17 and 258 are referred to as the *factors*.

Just as we used tables for the sum of any two single-digit numbers, we also can use tables for the product of any two single-digit numbers; from which we can with relative ease find the product of any two whole numbers. The process of finding the product of 17 and 258 is called multiplication and we say that we are multiplying 17 by 258.

In terms of the fill-in-the-blank format, we may represent a multiplication problem as:

(1) 6 X 4 = _____

(This means the sum of four 6's; that is, the fourth multiple of 6. So 6 X 4 means 6 + 6 + 6 + 6 or 24)

Factoring represents the "reverse" process. Namely, suppose we know the product of the two numbers and want to find what the two numbers are. That is we know the product but want to find the factors. For example:

(2) __ X __ = 24

The problem with (2) is that there are often several different answers. Namely:

6 X 4 = 24

8 X 3 = 24

12 X 2 = 24

24 X 1 = 24

To make this problem better defined we often give one of the two factors as well as the product. For example we might write:

(3) 6 X __ = 24

or

(4) 8 X __ = 24

The process of solving (3) or (4) is known as *division*.

The purpose of this module is to show how the processes of mutliplication, factoring, and division are developed in terms of place value numerals and how this provides us with a convenient way of dealing with large numbers.

Step 1:

View Videotape Lecture #3.

Step 2:

Read Module 3 of the Text.

Step 3:

When you understand the material in Steps 1 and 2, complete the following "Check-The-Main-Ideas" self-quiz by correctly filling in each blank.

Check The Main Ideas

The sum of four 8's is called the _____ multiple of 8. Since 8 + 8 + 8 + 8 = 32, the fourth multiple of 8 is _____. We write this as:

8 ___ 4 = 32

which we read as "8 _____ 4 is 32." 32 is called the _____ of 8 and 4 while 8 and 4 are referred to as _____ (of 32).

In Module 2 we talked about multiples of powers of _____. For example, the 8th multiple of 100 was simply an 8 followed by _____ 0's. We often have to find multiples of other numbers and that is what we do in this module. We start with single-digit numbers. The multiples of 0 and 1 are easy to find. For example the 24th multiple of 0 is _____ and the 24th multiple of 1 is _____. More generally, for any whole number n:

0 X n = ____

and

1 X n = ____

The _____ Property For Multiplication tells us that the product of two whole numbers doesn't depend on the order of the two factors. For example 8 X 4 = ___ X ____. That is, the fourth multiple of _____ is the same number as the _____ multiple of 4. In a similar way, since the 8th multiple of 10 is _____ the 10th multiple of 8 is _____. That is, the sum of ten 8's is _____.

fourth

32

X

times

product

factors

ten

two

0

24

0

n

Commutative

4;8

8

eighth

80

80

80

Since the sum of ten 8's is 80 and the sum of four 8's is 32, the sum of _____ 8's is 80 + 32 or 112. That is, 8 X 14 = _____.

fourteen

112

In essence we wrote 8 X 14 as 8 X (10 + 4) and then used the _____ property to rewrite 8 X (10 + 4) as (8 X 10) + (_____). If we had wanted to find the 347th multiple of 8, we could written:

distributive

8 X 4

8 X 347 = 8 X (300 + _____ + 7)

40

= (8 X 300) + (8 X 40) + (8 X ___)

7

The fact that 8 X 3 = 24 makes it relatively easy to compute 8 X 300. Namely 8 X 300 means 8 X (3 X 100); but by the associative property for multiplication 8 X (3 X 100) can be rewritten as (8 X 3) X _____. In place value notation we multiply a number by 100 by annexing 2 zeroes.

100

Hence 8 X 300 = (8 X 3) X 100

= 24 X 100

= _____

2,400

Since 8 X 4 = 32, 8 X 40 = _____. Hence the 347th multiple of 8 is given by:

320

8 X 347 = 8 X (300 + 40 + 7)

= (8 X 300) + (8 X 40) + (8 X 7)

= 2,400 + 320 + 56

= _____

2,776

In this sense multiplication is rapid addition because we have just computed the sum of 347 eights!

If we had wanted to find the sum of 347 eighty-sixes we'd have written:

86 X _____ *347*

or

(80 + __) X (300 + 40 + 7). *6*

The generalized form of the distributive property tells us now to multiply each term in the first set of parentheses by each term in the second set of parentheses. That is:

(80 + 6) X (300 + 40 + 7) =

(80 X 300) + (80 X 40) + (80 X ____) + *7*

(6 X 300) + (6 X ___) + (6 X 7) *40*

To multiply 80 by 300 we multiply 8 by 3 to get 24 and then annex _____ 0's. While *three*

this procedure may seem complicated, it's a lot easier than actually adding _____ 86's. *347*

Up to now we've started with the factors and found the product. But if we start with the product and want to find the factors, the process is known as _____. In terms of *factoring*

this vocabulary the fact that 24 is a multiple of 6 allows us to say that 6 is a _____ of 24. Since *factor*

27 is not a multiple of 6, 6 is not a _____ of 24. *factor*

The fact that 6 is a factor of 24 means that the blank in 6 X ___ = 24 can be replaced by a whole number. The operation by which we find the number we must multiply 6 by to get 24 is called _____. *division*

In the language of division we rewrite

6 X ___ = 24 as 24 ÷ 6 = _____. So the fact that

7 X 3 = 21, means that 21 ÷ 3 = _____ *7*

Let's look at a few examples. The first

6 multiples of 14 are 14, 28, 42, 56, ____, and *70*

_____. So 42 is the _____ multiple of 14 *84; 3rd*

and therefore 14 is a _____ of 42. This means *factor*

that 42 is _____ by 14. In fact since 14 X 3 = 42, *divisible*

we write that 42 ÷ 14 = _____. *3*

On the other hand, 47 is between 42 and 56.

Hence 14 is not a _____ of 47. Therefore 47 is *factor*

not _____ by 14. If we divide 47 by 14 the answer is *divisible*

between 3 and ____. That is 14 X 3 is less than 47 *4*

while 14 X 4 is greater than 47. The fact that

47 - 42 = 5 means that when we divide 47 by 14 the

_____ is 5. *remainder*

A key point is to remember that once you know

how to multiply, you automatically know how to

divide. For example the fact that 217 X 13 = 2,821

allows us to state that 2,821 ÷ ____ = 217 or that *13*

2,821 ÷ _____ = 13. *217*

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Step 4:

Do the Mastery Review:

Mastery Review

Answers:

1. At a price of $9 each, what is the cost of 4 handkerchiefs? 1. _____
2. What is the 5th multiple of 7? 2. _____
3. What is the 41st multiple of 2? 3. _____
4. How much is 9 X 7? 4. _____
5. How much is 0 X 7? 5. _____
6. How much is 78 X 1? 6. _____
7. There are 2 pints in a quart and 4 quarts in a gallon. How many pints are there in 5 gallons? 7. _____
8. What is the 10th multiple of 9? 8. _____
9. What is the 100th multiple of 78? 9. _____
10. How much is 7 X 3,000? 10. _____
11. Find the product of 700 and 3,000. That is, how much is 700 X 3,000? 11. _____
12. How much is 6,000,000 X 400,000? 12. _____
13. You buy 4 pencils for 9¢ each. Later you buy 3 more pencils at 9¢ each. How much did you pay altogether for the pencils? 13. _____
14. Use the distributive property to compute the value of 3 X (10 + 3). 14. _____
15. Find the 2nd multiple of 4,213. 15. _____
16. Find the 3rd multiple of 36. 16. _____
17. Find the 32nd multiple of 123. 17. _____
18. Is 7 a factor of 36? 18. _____
19. Is 9 a factor of 36? 19. _____
20. Is 15 a factor of 40? 20. _____
21. Is 9,102 divisible by 74? 21. _____

22. Is 6,400 divisible by 74? 22. _____

23. By what must we multiply 74 to obtain 7,622 as the product? 23. _____

Answers to Mastery Review

1. $36 2. 35 3. 82 4. 63

5. 0 6. 78 7. 40 8. 90

9. 7,800 10. 21,000 11. 2,100,000

12. 2,400,000,000,000 13. 63¢

14. 30 + 9 or 39 15. 8,426 16. 108

17. 3,936 18. no 19. yes

20. no 21. yes 22. no 23. 103

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Step 5:

Do Self-Test 3: Form A

(on the next page)

Self-Test 3: Form A

ANSWERS:

1. (a) List the first four multiples of 12.
   (b) List the first four multiples of 18.
   (c) What is the smallest (non-zero) whole number that has 12 and 18 as factors?

2. (a) How much is 76 X 4?
   (b) How much is 76 X 40?
   (c) How much is 40 X 7,600?
   (d) What is the 44th multiple of 76?

3. How much is 4 X 76 X 25?

4. (a) Find the product of 327 and 600?
   (b) How much is 327 X 599?

5. How much is 706 X 403?

6. (a) How much is (768 X 64) + (768 X 36)?
   (b) How much is 768 X (64 + 36)?
   (c) How much is (768 X 64) + 36?

7. (a) How much is 17,344 ÷ 542?
   (b) What number must we multiply by 542 to get 17,344 as the product?
   (c) Is 17,344 a multiple of 542?

8. (a) What is the quotient when 213,212 is divided by 706?
   (b) What is the remainder when 213,300 is divided by 706?
   (c) Is 213,300 a multiple of 706?

9. Find the quotient:

   $8{,}621\overline{)12{,}957{,}363}$

10. How much is:
   (a) (188,160 ÷ 28) ÷ 7?
   (b) 188,160 ÷ (28 ÷ 7)?
   (c) 188,160 ÷ (28 X 7)?

1. (a) _____
   (b) _____
   (c) _____

2. (a) _____
   (b) _____
   (c) _____
   (d) _____

3. _____

4. (a) _____
   (b) _____

5. _____

6. (a) _____
   (b) _____
   (c) _____

7. (a) _____
   (b) _____
   (c) _____

8. (a) _____
   (b) _____
   (c) _____

9. _____

10. (a) _____
   (b) _____
   (c) _____

(ANSWERS ARE ON THE NEXT PAGE)

Answers For Self-Test 3: Form A

1. *(a) 12, 24, 36, 48 (b) 18, 36, 54, 72 (c) 36*
2. *(a) 304 (b) 3,040 (c) 304,000 (d) 3,344*
3. *7,600*
4. *(a) 196,200 (b) 195,873*
5. *284,518*
6. *(a) 76,800 (b) 76,800 (c) 49,188*
7. *(a) 32 (b) 32 (c) Yes*
8. *(a) 302 (b) 88 (c) No*
9. *1,503*
10. *(a) 960 (b) 47,040 (c) 960*

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If you did each problem in Form A correctly, you may if you wish proceed to the next module. Otherwise continue with Step 6.

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Step 6:

Study the solutions to Self-Test 3 : Form A on the following pages, giving special emphasis to any problems you failed to answer correctly.

Solutions for Self-Test 3: Form A

1.

(a) In the language of multiplication the first four multiples of 12 are:

12 X 1, 12 X 2, 12 X 3, and 12 X 4;

that is:

12, 12 + 12, 12 + 12 + 12, and 12 + 12 + 12 + 12.

So we have:

12, 24, 36, and 48.

In essence, we're counting by 12's here.

(b) In this case we have:

18, 18 + 18, 18 + 18 + 18, and 18 + 18 + 18 + 18

Here we're counting by 18's.

or:

18, 36, 54, and 72.

(c) A number which has 12 as a factor is a multiple of 12--the first four of which are listed in part (a) of this problem. A number which has 18 as a factor is a multiple of 18--the first four of which are listed in part (b) of this problem. To be a (non-zero) multiple of 12 and 18, a number must belong to the lists in both (a) and (b). 36 fits this description. That is, 36 is the 3rd multiple of 12 and the 2nd multiple of 18.

12 X 18 or 216 is the 12th multiple of 18 and the 18th multiple of 12. But the problem asks for the smallest multiple.

Caution:

If you wrote 6 as the answer, you misinterpreted the problem. 6 is the biggest number that is a factor of both 12 and 18. What we want is the least common multiple of 12 and 18.

See how the vocabulary works? If both 12 and 18 are factors of a number, that number is a common multiple of 12 and 18.

2.

(a) 76 X 4 means the 4th multiple of 76, or:

76 + 76 + 76 + 76.

In vertical form we have:

```
  76  (first multiple of 76)
+ 76
 152  (second multiple of 76)
+ 76
 228  (third multiple of 76)
+ 76
 304  (fourth multiple of 76)
```

In principle this is much the same as Problem 1; but instead of counting by 12's or 18's, we're counting by 76's.

If we wish to use the multiplication table, we can rewrite the problem as:

(70 + 6) X 4 = 4 X (70 + 6) *by the commutative property for multiplication*

= (4 X 70) + (4 x 6) *by the distributive property*

= 280 + 24

= 304

4 X 70 = 4 X (7 X 10)
= (4 X 7) X 10
= 28 X 10
= 280

In vertical form this would be written as:

```
  2
  7 6
X 4
-----
304
```

(b) We may think of 76 X 40 as 76 X (4 X 10), which by the associative property is the same as (76 X 4) X 10. From part (a) we know that 76 X 4 is 304; and to multiply by 10 we annex a zero.

Hence:

76 X 40 = (76 X 4) X 10

= 304 X 10

= 3,040

In vertical form we often do this as:

That is:

hundreds	*tens*	*ones*
	7	*6*
X		*4*
	28	*24*
	2	
	30	*4*
3	*(0)*	*4*

If you have trouble placing the comma, first annex the 0 to get 3040. Then place the comma where it belongs.

2 (b) (continued)

$$\begin{array}{r} 7\ 6 \\ \times\ 4\ 0 \\ \hline 3\ 0\ 4\ 0 \end{array}$$

That is we multiply 76 by 4 and then "bring down" the 0.

We could use the fact that 6 X 0 = 0 and 7 X 0 = 0 to get:

$$\begin{array}{r} 7\ 6 \\ \times\ 4\ 0 \\ \hline 0\ 0 \\ 3\ 0\ 4 \\ \hline 3\ 0\ 4\ 0 \end{array}$$

(c) This is a generalization of what we did in part (b). Rewrite the problem, using the following sequence of steps:

$$40 \times 7{,}600 = (4 \times 10) \times (76 \times 100)$$

$$= (4 \times 76) \times (10 \times 100)$$

$$= (76 \times 4) \times 1{,}000$$

$$= 304 \times 1{,}000$$

$$= 304{,}000$$

We're using the commutative and associative properties of multiplication here.

We know that 76 X 4 = 304 from part (a); and to multiply by 1,000 we annex three 0's.

In summary, we multiplied 76 by 4 and then annexed the three 0's.

That is: $40 \times 7{,}600$ (1 zero, 2 3 zeros)

(d) The 44th multiple of 76 is 76 X 44. So we may proceed as follows:

$$76 \times 44 = 76 \times (40 + 4)$$

$$= (76 \times 40) + (76 \times 4)$$

$$= 3{,}040 + 304$$

$$= 3{,}344$$

In other words, we can solve this problem by using the results of parts (a) and (b).

In vertical form we have:

$$\begin{array}{r} 7\ 6 \\ \times\ 4\ 4 \\ \hline 3\ 0\ 4 \\ +\ 3\ 0\ 4\ 0 \\ \hline 3{,}3\ 4\ 4 \end{array}$$

(four 76's)
(forty 76's)
(forty-four 76's)

If we used the commutative property we'd have:

$$\begin{array}{r} 4\ 4 \\ \times\ 7\ 6 \\ \hline 2\ 6\ 4 \\ +\ 3\ 0\ 8 \\ \hline 3{,}3\ 4\ 4 \end{array}$$

(six 44's)
(seventy 44's)

2 (d) (continued)

However, our main purpose in giving this part of the problem is to show how the distributive property makes multiplication easier to do. That is, by the distributive property, we can reduce a complicated multiplication problem to a series of simpler ones. Namely:

Actually 76 X 44 is hardly a complicated problem, but is sufficient to illustrate the point we want to make.

$$76 \times 44 = 76 \times (40 + 4)$$

$$= (76 \times 40) + (76 \times 4)$$

$$= 3{,}040 + 304$$

$$= 3{,}344$$

We are now in a position to apply the results of parts (a) and (b)--both of which were relatively easy problems.

If you compare this approach with the vertical form, you'll notice that both say the same thing but in different formats.

Note that if we wanted to use only single digit multiples of the powers of ten, we could have written the problem as:

The key point is to understand the process of multiplication. It isn't too important which "recipe" you prefer to use.

$$(70 + 6) \times (40 + 4) =$$

$$(70 \times 40) + (70 \times 4) + (6 \times 40) + (6 \times 4) =$$

$$2{,}800 + 280 + 240 + 24 =$$

$$3{,}344$$

3.

We find the product of any number of factors by computing products of two factors at a time. For example by the associative property of multiplication we have:

$$4 \times 76 \times 25 = (4 \times 76) \times 25$$
$$= 304 \times 25$$
$$= 304 \times (20 + 5)$$
$$= (304 \times 20) + (304 \times 5)$$
$$= 6{,}080 + 1{,}520$$
$$= 7{,}600$$

We know that 4 X 76 = 304 from the previous problem.

In vertical form:

$$\begin{array}{r} 304 \\ \times \quad 25 \\ \hline 1520 \\ +\ 6080 \\ \hline 7{,}600 \end{array}$$

(five 304's)

(twenty 304's)

The fact that we can rearrange and regroup the factors can give us a quicker way of getting the product. For example, you may have noticed that 4 X 25 = 100 and that it's easy to multiply by a hundred. In this case, you could write:

$$4 \times 76 \times 25 = (4 \times 76) \times 25$$
$$= (76 \times 4) \times 25$$
$$= 76 \times (4 \times 25)$$
$$= 76 \times 100$$
$$= 7{,}600$$

To multiply by 100 we annex two 0's.

4.

(a) To find the product of 327 and 600, we can find the product of 327 and 6; and then annex two 0's. So we have:

327 X 600 = 327 X (6 X 100)
= (327 X 6) X 100

4. (a) (continued)

$327 \times 6 = (300 + 20 + 7) \times 6$

$= (300 \times 6) + (20 \times 6) + (7 \times 6)$

$= 1{,}800 + 120 + 42$

$= 1{,}962$

We're again using the distributive property.

The tables tell us that $3 \times 6 = 18$, $2 \times 6 = 12$ and $7 \times 6 = 42$.

(In vertical form:

$$\begin{array}{r} 1\ 4 \\ 3\ 2\ 7 \\ \times \quad\ 6 \\ \hline 1\ 9\ 6\ 2 \end{array}$$

That is:

hundreds	*tens*	*ones*
3	*2*	*7*
	X	*6*
18	*12*	*42*
18	*16*	*2*
19	*6*	*2*

Therefore:

$327 \times 600 = (327 \times 6) \times 100$

$= 1962 \times 100$

$= 196200$

$= 196{,}200$

In vertical form:

$$\begin{array}{r} 3\ 2\ 7 \\ \times \ 6\ 0\ 0 \\ \hline 1\ 9\ 6\ 2\ 0\ 0 \end{array}$$

(b) We could proceed just as we did in part (a). Namely:

$327 \times 599 = 327 \times (500 + 90 + 9)$

$= (327 \times 500) + (327 \times 90) + (327 \times 9)$

$= 163{,}500 + 29{,}430 + 2{,}943$

$= 195{,}873$

$$\begin{array}{r} 3 \\ 327 \\ \times\ 5 \\ \hline 1635 \end{array} \qquad \begin{array}{r} 6 \\ 327 \\ \times\ 9 \\ \hline 2943 \end{array}$$

But if we understand addition and subtraction there is a quicker way of doing (b). Use the fact that 599 is 1 less than 600. That is:

$327 \times 599 = 327 \times (600 - 1)$

$= (327 \times 600) - (327 \times 1)$

$= 196{,}200 - 327$

$= 195{,}873$

Think in terms of money. If you have 600 checks for \$327 each and you tear up one of the checks you have 599 checks at \$327 each. So:

$$\begin{array}{rl} & 600 \text{ @\$327} = \$196{,}200 \\ - & \ \ 1 \text{ @\$327} = (-)\ \ 327 \\ \hline & 599 \text{ @\$327} = \$195{,}873 \end{array}$$

4. (concluded)

Rounding-Off Note:

If we had been given the problem 327 X 599 without having been given part (a) first, we should recognize that 327 X 600 is a "little" too big to be the correct answer. This tells us that the answer is close to (but a bit less than) 327 X 600 which is easier to compute than 327 X 599. In other words it is relatively easy to conclude that the product is a little less than 196,200 and this helps guard against very large errors.

In fact it is exactly 327 too big.

A quicker estimate (but less accurate) occurs if round off 327 to 300 and 599 to 600. We'd then get 300 X 600 which is 18 followed by four 0's or 180,000.

This type of estimate is helpful even when we use calculators.

5.

We have:

$$706 \times 403 = 706 \times (400 + 3)$$

$$= (706 \times 400) + (706 \times 3)$$

$$= 282{,}400 + 2{,}118$$

$$= 284{,}518$$

706 X 4 = (700 + 6) X 4
= (700 X 4) + (6 X 4)
= 2,800 + 24
= 2,824

706 X 3 = (700 + 6) X 3
= (700 X 3) + (6 X 3)
= 2,100 + 18
= 2,118

In vertical form we have:

```
   7 0 6
 X 4 0 3
 -------
   2 1 1 8   (three 706's)
 0 0 0 0 0   (just place holders)
 2 8 2 4 0 0 (four-hundred 706's)
 -----------
 2 8 4,5 1 8
```

You may omit the row of 0's by indenting the next row two places. That is:

```
   7 0 6
 X 4 0 3
 -------
   2 1 1 8
 2 8 2 4
 -------
 2 8 4,5 1 8
```

Note that a very common error is to ignore the 0 in 403. That is, people sometimes write:

```
   7 0 6
 X 4 0 3
 -------
   2 1 1 8
 2 8 2 4
 -------
 3 0,3 5 8
```

But 706 X 403 is greater than 700 X 400, which is 280,000. In other words, by rounding off, we should recognize that the correct answer must be greater than 280,000; thus excluding 30,358 as a possibility.

6.

Before we start this problem, keep in mind that our aim is to show how we may and may not use grouping symbols to our advantage. Remember that everything within a pair of parentheses is treated as one number. Hence we do the arithmetic inside the parentheses before we remove them.

(a) $768 \times 64 = 768 \times (60 + 4)$

$= (768 \times 60) + (768 \times 4)$

$= 46{,}080 + 3{,}072$

$= 49{,}152$

$768 \times 36 = 768 \times (30 + 6)$

$= (768 \times 30) + (768 \times 6)$

$= 23{,}040 + 4{,}608$

$= 27{,}648$

Therefore:

$(768 \times 64) + (768 \times 36) =$

$49{,}152 + 27{,}648 =$

$76{,}800$

(b) Doing what's inside the parentheses first we have:

$768 \times (64 + 36) =$

$768 \times 100 =$

$76{,}800$

$768 \times 4 =$
$(700 + 60 + 8) \times 4 =$
$(700 \times 4) + (60 \times 4) + (8 \times 4) =$
$2{,}800 + 240 + 32 =$
$3{,}072$

$768 \times 6 =$
$(700 + 60 + 8) \times 6 =$
$(700 \times 6) + (60 \times 6) + (8 \times 6) =$
$4200 + 360 + 48 =$
4608

$768 \times 3 =$
$(700 + 60 + 8) \times 3 =$
$(700 \times 3) + (60 \times 3) + (8 \times 3) =$
$2100 + 180 + 24 =$
2304

It is no coincidence that parts (a) and (b) have the same answer. Part (b) is a quicker way of doing part (a). Namely, the the distributive property, $768 \times (64 + 36)$ means the same as the expression: $(768 \times 64) + (768 \times 36)$

6. (continued)

(c)

In part (b) it might be tempting to omit the parentheses. If we did the problem would look like:

768 X 64 + 36

Suppose we now grouped the terms as:

(768 X 64) + 36

This would give us the problem presented in part (c). Doing the arithmetic within the parentheses first we get:

(768 X 64) + 36 =

49,152 + 36 =

49,188

Key Point

Parts (b) and (c) would both look like 768 X 64 + 36 if the parentheses were omitted. The fact that we get different answers in the two parts means that it is important to keep the parentheses.

Looks can be deceiving! Parts (b) and (c) look more alike than do parts (b) and (a). Yet (b) and (a) mean the same thing while (b) and (c) are very different.

This is what's nice about the associative properties of addition and multiplication. It makes no difference whether we write 2 + (3 + 4) or (2 + 3) + 4 and as a result we may omit the parentheses and simply write 2 + 3 + 4.

7.

(a) 17,344 ÷ 542 means the same thing as

$542\overline{)17,344}$

The following facts will now prove helpful: *542 X 1 = 542*

542 X 2 = 1,084

542 X 3 = 1,626

Notice the change in order when we rewrite the problem:

17,344 ÷ 542

$542\overline{)17,344}$

7. (a) (continued)

So we have:

$$\begin{array}{r} \quad\;\; 3\;2 \\ 5\;4\;2\;\overline{)\,1\;7\;3\;4\;4} \\ -\;1\;6\;2\;6 \\ \hline \quad\;\; 1\;0\;8\;4 \\ -\;1\;0\;8\;4 \\ \hline \end{array}$$

Notice the placement of the digits in the quotient. 542 is less than 1, 17, and 173 so the first digit in the quotient (3) is placed over the 4 in 1734.

Step by step we have:

$$\begin{array}{r} 3 \\ 542\overline{)17344} \\ -1626 \\ \hline 108 \end{array}$$

108 is less than 542 so we know that 3 is the largest multiple of 542 we can use.

$$\begin{array}{r} 3 \\ 542\overline{)17344} \\ -1626 \\ \hline 1084 \end{array}$$

$$\begin{array}{r} 32 \\ 542\overline{)17344} \\ -1626 \\ \hline 1084 \\ -1084 \\ \hline 0 \end{array}$$

Check:

$$\begin{array}{r} 5\;4\;2 \\ \times\;3\;2 \\ \hline 1\;0\;8\;4 \\ 1\;6\;2\;6 \\ \hline 1\;7{,}3\;4\;4 \end{array}$$

(b) This is simply a restatement of part (a). That is, 17,344 ÷ 542 means the number we must multiply by 542 to get 17,344 as the product. In terms of fill-in-the-blank, we're asking:

$$542 \times \_\_\_ = 17{,}344$$

Don't confuse this with the problem of asking for the product of 542 and 17,344. That is, we're asking:

542 X _____ = 17,344

not

542 X 17,344 = _____

In fact, we may "revisit" part (a) to see:

$$\begin{array}{r} 542\overline{)17{,}344} \\ -16{,}260 \quad \textit{(thirty 542's)} \\ \hline 1{,}084 \\ -\;1{,}084 \quad \textit{(two 542's)} \\ \hline 0 \end{array}$$

From another point of view:

$$\begin{array}{r} 542 \times 30 = 16{,}260 \\ 542 \times\;\;2 = \;\;1{,}084 \\ \hline 542 \times 32 = 17{,}344 \end{array}$$

Recall that the distributive property tells us that 542 X 32 = (542 X 30) + (542 X 2)

(Thirty 542's and two 542's is thirty-two 542's)

7. (concluded)

(c) In parts (a) and (b), which were different forms of the same question, we saw that 17,344 is the 32nd multiple of 542. In particular, then, we've already shown that 17,344 is a multiple of 542.

In other words, if we counted by 542's, the 32nd number in our list would be 17,344.

Alternative wordings:

To say that 17,344 is a multiple of 542 means the same thing as saying that 542 is a factor of 17,344 or that 17,344 is divisible by 542.

8.

(a) To find the quotient when 213,212 is divided by 706 means that we want to divide 213,212 by 706. That is: $706\overline{)213,212}$. In a step-by-step way we have:

```
        3
706)213212
  -2118
    14
```

706 X 3 =
(700 + 6) X 3 =
(700 X 3) + (6 X 3) =
2,100 + 18 =
2,118

```
        3
706)213212
  -2118
   141
```

Everytime we bring a digit down from the dividend, a digit must go above it in the qotient. So make sure that you put the 0 above the 1 as a place holder.

```
       30
706)213212
  -2118
   - 141
     000
    1412
```

Sometimes it's easier to write a short table before you start the problem. For example:

706 X 1 = 706
706 X 2 = 1,412
706 X 3 = 2,118 etc

You never have to beyond 706 X 9.

```
       302
706)213212
  - 2118
    141
  - 000
    1412
  - 1412
       0
```

8. (a) (continued)

What we were actually doing in this division problem was:

$$\begin{array}{r} 213,212 \\ -\ 211,800 \\ \hline 1,412 \\ -\ 1,412 \\ \hline 0 \end{array}$$

(3 hundred 706's)

(two 706's)

So 213,212 is the 302nd multiple of 706.

Check:

$$\begin{array}{r} 706 \\ \times\ 302 \\ \hline 1412 \\ +\ 000 \\ \hline 2118 \\ \hline 213212 \end{array}$$

Note:

If you had misplaced the 2 in the quotient to get 32, the check wouldn't have worked. Namely:

$$\begin{array}{r} 706 \\ \times\ 32 \\ \hline 1412 \\ 2118 \\ \hline 22592 \end{array}$$

(that is; 22,592)

which is much less than 213,212

In fact by rounding off, 706 X 32 is a bit greater than 700 X 30 or 21,000. From this we can see that the product is nowhere near 213,212.

(b)

We could actually compute $706\overline{)213,300}$ but if we use the result of part (a) there is a quicker way. Namely we already know that the 302nd multiple of 706 is 213,212. Hence:

$$\begin{array}{r} 213,300 \\ -\ 213,212 \\ \hline 88 \end{array}$$

(302nd multiple of 706)

If we hadn't already done part (a) we'd have written:

$$\begin{array}{r} \quad 302\ R88 \\ 706\overline{)213,300} \\ 211\ 8 \\ \hline 1\ 50 \\ -\ 0\ 00 \\ \hline 1\ 500 \\ 1\ 412 \\ \hline 88 \end{array}$$

(c) We have just seen that 213,300 is not divisible by 706. Hence 213,300 is not a multiple of 706. More specifically, the 302nd multiple of 706 is 213,212 and the 303rd multiple of 706 is 213,918. So 213,300 is between the 302nd and 303rd multiple of 706.

If it had been divisible, the remainder would have been 0.

That is, the 303rd multiple of 706 is 706 more than the 302nd multiple of 706. 213,212 + 706 = 213,918. As a check, 303 X 706 = 213,918

9.

This problem is more cumbersome than the previous division problems but it follows exactly the same principles. To make things easier later, begin by writing the first "few" multiples of 8,621. These are shown in the margin. The division "recipe" then gives us:

```
                1503
8621)12957363
    -8621
     43363
    -43105
       2586
     - 0000
       25863
     - 25863
           0
```

8,621 X 1 = 8621
8,621 X 2 = 17242
8,621 X 3 = 25863
8,621 X 4 = 34484
8,621 X 5 = 43105
8,621 X 6 = 51726
8,621 X 7 = 60347
8,621 X 8 = 68968
8,621 X 9 = 77589
(We're counting by 8,621's but never have to go beyond the 9th multiple because of place value)

What we have shown is that 12,957,363 is the 1,503rd multiple of 8,621. In other words, if we were to count by 8,621's the 1,503rd number we'd come to is 12,957,363.

Check:

```
          8 6 2 1
        X 1 5 0 3
        2 5 8 6 3 (three 8,621's)
        0 0 0 0 0 (place holder)
  4 3 1 0 5 0 0 (5 hundred 8,621's)
  8 6 2 1 0 0 0 (1 hundred 8,621's)
1 2 9 5 7 3 6 3
```

or:

```
  12,957,363
- 8,621,000  (1,000 - 8,621's)
  4,336,363
- 4,310,500  (  500 - 8,621's)
     25,863
-   25,863  (    3 - 8,621's)
          0
            (1,503 - 8,621's)
```

See How powerful place value notation is!

8,621 X 1 = 8621
8,621 X 2 = 17242
8,621 X 3 = 25863
8,621 X 4 = 34484
8,621 X 5 = 43105
8,621 X 6 = 51726
8,621 X 7 = 60347
8,621 X 8 = 68968
8,621 X 9 = 77589

```
        1
8621)12957363
   -8621
    4336

       15
8621)12957363
   - 8621
     43363
   - 43105
       258

      150
8621)12957363
    -8621
     43363
   - 43105
      2586
    - 0000
      2586

      1503
8621)12957363
    -8621
     43363
   -43105
      2586
    - 0000
      25863
   - 25863
```

10.

(a) We do the arithmetic within the parentheses first. Namely:

188,160 ÷ 28 means

$$
\begin{array}{r}
6720 \\
28\overline{)188160} \\
-168 \\
\hline
201 \\
-\ 196 \\
\hline
56 \\
-56 \\
\hline
00 \\
-\ 00 \\
\hline
0
\end{array}
$$

28 X 1 = 28; 28 X 2 = 56
28 X 3 = 84; 28 X 4 = 112
28 X 5 = 140; 28 X 6 = 168
28 X 7 = 196; 28 X 8 = 224
28 X 9 = 252

Don't forget the 0 in the quotient.

Remember that 672 X 28 is less than 700 X 30 or 21,000. Hence 672 X 28 cannot possibly equal 188,160!

Therefore:

(188,160 ÷ 28) ÷ 7 =

6,720 ÷ 7 =

$$
\begin{array}{r}
960 \\
7\overline{)6720} \\
-63 \\
\hline
42 \\
-42 \\
\hline
00 \\
-\ 00 \\
\hline
0
\end{array}
$$

Check:

960 X 7 =
(900 + 60) X 7 =
(900 X 7) + (60 X 7) =
6,300 + 420 =
6,720

Hence (188,160 ÷ 28) ÷ 7 = 960

As an application, suppose I am one of 28 people who buy a company for $188,160. Then each of the 28 people pays $6,720 if the cost is equally shared. Now suppose I have 6 partners who share my cost equally. Then each of us seven must pay $960.

Division is not associative, so grouping makes a big difference as we shall now see.

10. (continued)

(b)

Notice that we have regrouped the numbers that appeared in part (a). That is, both (a) and (b) would look like 188,160 ÷ 28 ÷ 7 if the grouping symbols (parentheses) had been omitted.

Again we work within the parentheses first. Since 28 ÷ 7 = 4, we have:

188,160 ÷ (28 ÷ 7) =

188,160 ÷ 4 =

```
   47040
4)188160
 -16
  28
 -28
  01
 - 00
   16
 - 16
    00
  - 00
     0
```

```
  188,160
- 160,000  (40,000   4's)
  28,160
- 28,000  ( 7,000   4's)
     160
   - 160  (     40   4's)
       0  (4,7040   4's)
```

There is a big difference between paying $960 for a share and paying $47,040 as a share. Hence keeping track of the parentheses is very important in a division problem.

So while (a) and (b) may look a lot alike they are really quite different. On the other hand, (a) and (c) look less alike, but as we shall soon see, they mean the same thing.

10.

(c)

Doing the arithmetic within the parentheses first we see that:

$$28 \times 7 = (20 + 8) \times 7$$
$$= (20 \times 7) + (8 \times 7)$$
$$= 140 + 56$$
$$= 196$$

Hence:

$$188{,}160 \div (28 \times 7) =$$

$$188{,}160 \div 196 =$$

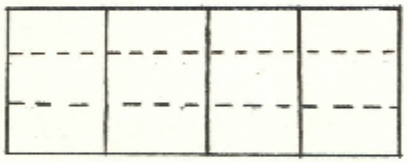
$$\begin{array}{r} 960 \\ 196\overline{)188160} \\ -1764 \\ \hline 1176 \\ -1176 \\ \hline 0 \end{array}$$

Notice why (a) and (c) are related. For the sake of illustration, we'll use smaller numbers. Suppose we divide a pie into 4 equal parts and we then divide each of these four parts into 3 equal parts. Then altogether we've divided the pie into 12 equal parts. That is, if we divide a number by 4 and then divide this quotient by 3, it is the same as if we had divided originally by 4 X 3.

In a similar way, to divide by 28 and then by 7 is the same as dividing by 28 X 7 or 196.

196 X 1 = 196
196 X 2 = 392
196 X 3 = 588
196 X 4 = 784
196 X 5 = 980
196 X 6 = 1176
196 X 7 = 1372
196 X 8 = 1568
196 X 9 = 1764

$$\begin{array}{rl} 188{,}160 & \\ -\underline{176{,}400} & (900 - 196\text{'s}) \\ 11{,}760 & \\ -\underline{11{,}760} & (60 - 196\text{'s}) \\ 0 & (960 - 196\text{'s}) \end{array}$$



In terms of the application mentioned at the end of part (a), $360 is the share each person would pay if 196 people equally shared the cost of the $188,160 company.

Step 7:

Do Self-Test 3: Form B

Self-Test 3: Form B

1. (a) List the first four multiples of 16.
   (b) List the first four multiples of 24.
   (c) What is the smallest (non-zero) whole number that has 16 and 24 as factors?

2. (a) How much is 82 X 6?
   (b) How much is 82 X 60?
   (c) How much is 8,200 X 600?
   (d) What is the 66th multiple of 82?

3. How much is 6 X 82 X 50?

4. (a) What is the product of 821 and 500?
   (b) How much is 821 X 499?

5. How much is 809 X 504?

6. (a) How much is (453 X 49) + (453 X 51)?
   (b) How much is 453 X (49 + 51)?
   (c) How much is (453 X 49) + 51?

7. (a) How much is 34,722 ÷ 643?
   (b) What number must we multiply by 643 to get 34,722 as the product?
   (c) Is 34,722 a multiple of 643?

8. (a) What is the quotient when 204,624 is divided by 504?
   (b) What is the remainder when 205,000 is divided by 504?
   (c) Is 205,000 a multiple of 504?

9. Find the quotient:

   $7{,}543\overline{)17{,}356{,}443}$

10. How much is:
   (a) (138,720 ÷ 24) ÷ 4?
   (b) 138,720 ÷ (24 ÷ 4)?
   (c) 138,720 ÷ (24 X 4)?

ANSWERS:

1. (a) _____
   (b) _____
   (c) _____

2. (a) _____
   (b) _____
   (c) _____
   (d) _____

3. _____

4. (a) _____
   (b) _____

5. _____

6. (a) _____
   (b) _____
   (c) _____

7. (a) _____
   (b) _____
   (c) _____

8. (a) _____
   (b) _____
   (c) _____

9. _____

10. (a) _____
   (b) _____
   (c) _____

(ANSWERS ARE ON NEXT PAGE)

Answers for Self-Test 3: Form B

1. *(a) 16, 32, 48, 64 (b) 24, 48, 72, 96 (c) 48*
2. *(a) 492 (b) 4,920 (c) 4,920,000 (d) 5,412*
3. *24,600*
4. *(a) 410,500 (b) 409,679*
5. *407,736*
6. *(a) 45,300 (b) 45,300 (c) 22,248*
7. *(a) 54 (b) 54 (c) Yes*
8. *(a) 406 (b) 376 (c) No*
9. *2,301*
10. *(a) 1,445 (b) 23,120 (c) 1,445*

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If you did each problem in Form B correctly, you may, if you wish, proceed to the next module. Otherwise, continue with Step 8.

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Step 8:

View the solutions for Self-Test 3: Form B on Videotape Lecture 3 S. Pay special attention to the solutions of those problems for which you failed to get the correct answers. *Feel free to rewind the tape at any time to restudy any probleme that give you difficulty.*

Step 9:

Do Self-Test 3: Form C.

Self-Test 3: Form C

ANSWERS:

1. (a) List the first four multiples of 14.
   (b) List the first four multiples of 21.
   (c) What is the smallest (non-zero) whole number that has 14 and 21 as factors?

2. (a) How much is 67 X 3?
   (b) How much is 67 X 30?
   (c) How much is 670 X 3,000?
   (d) What is the 33rd multiple of 67?

3. How much is 8 X 67 X 25?

4. (a) What is the product of 937 and 400?
   (b) How much is 937 X 399?

5. How much is 507 X 803?

6. (a) How much is (923 X 72) + (923 X 28)?
   (b) How much is 923 X (72 X 28)?
   (c) How much is (923 X 72) + 28?

7. (a) How much is 22,833 ÷ 531?
   (b) What number must we multiply by 531 to get 22,833 as the product?
   (c) Is 22,833 a multiple of 531?

8. (a) What is the quotient when 102,921 is divided by 507?
   (b) What is the remainder when 103,000 is divided by 507?
   (c) Is 103,000 a multiple of 507?

9. Find the quotient:

   $2{,}643\overline{)37{,}081{,}290}$

10. How much is:
   (a) (194,040 : 42) : 6?
   (b) 194,040 : (42 ÷ 6)?
   (c) 194,040 ÷ (42 X 6)?

ANSWERS:

1. (a) _____
   (b) _____
   (c) _____

2. (a) _____
   (b) _____
   (c) _____
   (d) _____

3. _____

4. (a) _____
   (b) _____

5. _____

6. (a) _____
   (b) _____
   (c) _____

7. (a) _____
   (b) _____
   (c) _____

8. (a) _____
   (b) _____
   (c) _____

9. _____

10. (a) _____
   (b) _____
   (c) _____

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 3: Form C

1. (a) 14, 28, 42, 56 (b) 21, 42, 63, 84 (c) 42
2. (a) 201 (b) 2,010 (c) 2,010,000 (d) 2,211
3. 13,400
4. (a) 374,800 (b) 373,863
5. 407,121
6. (a) 92,300 (b) 92,300 (c) 66,484
7. (a) 43 (b) 43 (c) Yes
8. (a) 203 (b) 79 (c) No
9. 14,030
10. (a) 770 (b) 27,720 (c) 770

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THIS CONCLUDES OUR STUDY GUIDE PRESENTATION FOR MODULE #3.

HOPEFULLY, YOU WILL NOW FEEL READY TO BEGIN MODULE #4.

HOWEVER, IF YOU STILL FEEL UNCERTAIN OF THE MATERIAL IN THIS MODULE YOU SHOULD CONSULT A TEACHER, A FRIEND, OR A FELLOW-STUDENT FOR ADDITIONAL REINFORCEMENT.

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